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Analytical and numerical calculation of the buckling stress for panels with closed-section stiffeners

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Abstract

The forthcoming second generation of EN 1993-1-5 is bringing changes in terms of buckling and interpolation curves that allow the positive effects of torsional stiffness of closed-section stiffeners to be taken into account. Regarding the determination of the critical buckling stress for global plate buckling, analytical or numerical methods may be used. FprEN 1993-1-5 provides a simplified analytical equation for determining the critical buckling stress of an equivalent orthotropic plate, which can also be found in the informative Annex A of EN 1993-1-5. The given equation does not take into account the torsional stiffness of longitudinal stiffeners with closed cross-sections. The latter are often used in plated structures due to fabrication and strength benefits. This paper compares two methods for determining the elastic critical plate buckling stress that consider the positive effect of the torsional stiffness of closed-section stiffeners. First, a newly proposed analytical method that accounts for the torsional stiffness of closed-section stiffeners is derived on the same basis as the current equation in Annex A. Secondly, a numerical linear buckling analysis is performed on a large number of stiffened panels. The advantages and disadvantages of both methods are pointed out. Finally, both values are used to determine the ultimate resistance of plates and with GMNIA results from previous studies.

Keywords

Plate buckling, plate-like behaviour, stiffened panel, longitudinal stiffeners, torsional stiffness of stiffener, critical buckling stress

1 Introduction

The design methodology for stiffened plates in EN 1993-1-5 [1] distinguishes between three types of buckling behaviour in stiffened plates: column-like behaviour without any post-buckling resistance, plate-like behaviour considering post-buckling resistance, and interactive behaviour between the two. Based on the elastic critical buckling stresses for column-like buckling $\sigma_{cr,c}$ and plate-like buckling $\sigma_{cr,p}$, the reduction factors χ_c and ρ_p are determined using the European buckling curves and the Winter formula, respectively. The final reduction factor for the interactive behaviour results from the reduction factors and the ratio between the behaviour modes $\sigma_{cr,p}/\sigma_{cr,c}$, from which the load-bearing capacity of the stiffened plate is determined. In recent years, several investigations have shown that when considering plates with closed-section stiffeners, the application of the design rules [1] can lead to unsafe results if the torsional stiffness of the stiffeners is taken into account in the critical stress calculation [2,3]. The current Amendment A2 to EN 1993-1-5 [1] therefore stipulates that the torsional stiffness of the stiffeners should generally be neglected when determining the critical buckling stresses. However, neglecting the beneficial influence of torsional stiffness can lead to a significant underestimation of plate resistance [4] and thus to an increase in construction costs.

In order to improve the economy of design of plates stiffened with closed-section stiffeners and loaded in pure compression, Kövesdi et al. [5] and Pourostad and Kuhlmann [6] have recently proposed alternative design procedures that allow the positive effects of torsional stiffness on plate resistance to be taken into account. Based on the latter [6], the forthcoming second generation of EN 1993-1-5, namely FprEN 1993-1-5 [7], will bring changes in terms of buckling and interpolation curves that allow to consider the positive effects of torsional stiffness of closed-section stiffeners.

For the determination of the elastic critical buckling stress for plate-like buckling, namely $\sigma_{cr,p}$, any relevant method can be used, including linear buckling analysis (LBA) with Finite Element methods, dedicated software such as EBPlate [8] or the analytical formula in Annex A of EN 1993-1-5 [1]. In the numerical analysis, the torsional stiffness of trapezoidal stiffeners is considered directly in the model. However, the analytical equation for the elastic

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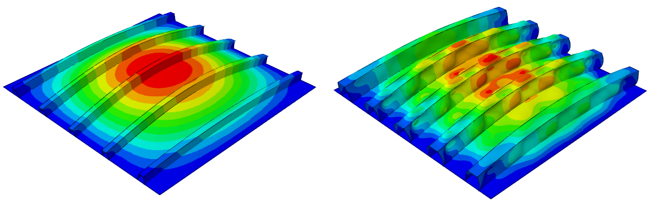
critical plate buckling stress of an equivalent orthotropic plate in the informative Annex A does not take into account the torsional stiffness of longitudinal stiffeners with closed cross-section. Neglecting the torsional stiffness in the calculation of $\sigma_{cr,p}$ leads to conservative results when the newly proposed interpolation equation according to [7] is applied. Therefore, in the first part of this paper, a new analytical equation for the estimation of the critical buckling stress for plate-type buckling is presented, which takes into account the torsional inertia of the stiffeners and is consistent with the existing equation from Annex A, EN 1993-1-5.

The critical elastic buckling stress for plate-like behaviour of a stiffened plate ($\sigma_{cr,p}$) corresponds to a "global" buckling mode in which any "local" buckling of subpanels is ignored. An example of a perfectly "global" buckling mode is shown in Figure 1(a), while Figure 1(b) shows an example of a "global" mode combined with "local" buckling of subpanels. For the majority of stiffened plate configurations, especially for plates with relatively rigid stiffeners, it is difficult to find "clear" buckling modes that represent a perfectly global buckling mode. In the second part of the paper, a FE linear buckling analysis is presented where the first global eigenmode was identified for a variety of longitudinally stiffened plates with different geometric parameters.

Finally, both approaches to determine $\sigma_{cr,p}$, namely the newly proposed analytical approach and the numerical approach, are used to calculate the ultimate resistance of plates according to the new procedure from FprEN 1993-1-5 [7] and compared with the GMNIA results from [6].



(a) A "global" buckling mode (b) A combined buckling mode

Figure 1 Examples of buckling modes resulting from a linear buckling analysis (LBA) for a longitudinally stiffened plate loaded in compression

2 Analytical determination of $\sigma_{cr,p}$

2.1 Neglecting torsional rigidity of closed longitudinal stiffeners

The analytical equation for the elastic critical buckling stress of a stiffened plate given in Annex A of EN 1993-1-5 is based on an orthotropic plate approach, where a structurally orthotropic plate is reduced to a materially orthotropic plate with elastic properties equal to the average properties of the original plate. Thus, instead of a stiffened plate, a homogeneous plate of constant thickness is considered that has the same stiffness characteristics as the stiffened plate. This procedure requires the determination of three elastic rigidity constants, namely D_x, D_y and D_{xy}, which represent the equivalent flexural rigidities of structurally orthotropic plates. According to Timoshenko and Woinowski-Krieger [9], the flexural rigidities for a plate reinforced with a series of equidistant ribs in the longitudinal direction x, can be determined as follows:

$$D_x = \frac{EI_{sl}}{b} \tag{1}$$

$$D_y = H = D_{xy} = \frac{EI_p}{b} \tag{2}$$

where I_{sl} is the second moment of area of the entire stiffened plate and I_p is the second moment of area for the bending of the plate.

For a simply supported, longitudinally stiffened plate loaded in axial compression (N_x = $-p_x$ and N_y = N_{xy} = 0), the governing differential equation of an orthotropic plate, also known as "Huber's equation", can be written:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = -p_x \frac{\partial^2 w}{\partial x^2} \tag{3}$$

Eq. (3) is a homogeneous partial differential equation and the critical buckling load is obtained from its solution. The following displacement function solves the equation for simply supported boundary conditions:

$$w = C_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{4}$$

By introducing nondimensional parameters for the relative axial stiffness δ and the relative flexural stiffness γ in Eqs. (5,6) and minimizing the solution of the differential equation (3), the buckling coefficient $k_{\sigma,p}$ from Eq. (7) can be defined in two branches:

$$\delta = A_{sl} / A_p \tag{5}$$

$$\gamma = I_{sl} / I_p \tag{6}$$

$$\sigma_{cr,p} = \sigma_E k_{\sigma,p} \tag{7}$$

$$\begin{aligned} k_{\sigma,p} &= \frac{(1+\alpha^2)^2 + \gamma - 1}{\alpha^2 (1+\delta)} \quad \text{for } \alpha \le \sqrt[4]{\gamma} \\ k_{\sigma,p} &= \frac{2(1+\sqrt{\gamma})}{(1+\delta)} \quad \text{for } \alpha > \sqrt[4]{\gamma} \end{aligned} \tag{8}$$

where α is the aspect ratio of the plate and A_{sl} and A_p are the sum of the gross areas of the individual longitudinal stiffeners and the gross area of the plate, respectively.

The solution (8) is the plate buckling coefficient for the global buckling of the stiffened plate under uniform compression given in Annex A of EN 1993-1-5.

2.2 Considering torsional rigidity of closed longitudinal stiffeners

In this section, the analytical equation for $\sigma_{cr,p}$ presented in section 2.1 is modified to also take into account the torsional inertia of the stiffeners.

According to Saint-Venant's theory and Bredt's formula for thin-walled cross-sections with a closed shape, the torsional rigidity of the plate stiffened with closed ribs can be calculated by the following equation:

$$2H = G\frac{n_{st}}{b}I_t = G\frac{n_{st}}{b}\frac{4A_R^2}{\sum\frac{ds_i}{t_{s,i}}} \tag{9}$$

where 2H is the effective torsional rigidity of an orthotropic plate from Eq. (3). I_t is the torsional section constant of the closed-rib stiffened plate, G is the shear modulus, b is the plate width, n_{st} is the number of longitudinal stiffeners, A_R is the average of the areas enclosed by the outer and inner boundaries of the cross-section of the closed-section stiffener and ds_i is the length of the cross-sectional part with thickness $t_{s,I}$, see Figure 2.

Introducing the nondimensional parameter for the relative torsional stiffness and assuming that the flexural rigidities in both perpendicular directions remain the same as in section 2.1, the solution to Eq. (5) can be written as follows:

$$\theta = \frac{n_{st}GI_t}{bD} \tag{10}$$

$$\sigma_{cr,p} = \sigma_E k_{\sigma,p} = \frac{\pi^2 Et^2}{12(1-\nu^2)b^2}\frac{1}{(1+\delta)}\left[\left(\frac{m}{\alpha}\right)^2\gamma + \left(\frac{\alpha}{m}\right)^2 + \theta n^2\right] \tag{11}$$

If we assume a buckling shape with one half-wave in the transverse direction (n = 1), the buckling coefficient $k_{\sigma,p}$ is a function of m, α and the nondimensional parameters δ, γ and θ of the stiffener. Finally, by minimizing the function (11), $k_{\sigma,p}$ can be defined in two branches, the first part corresponding to the value of $k_{\sigma,p}$ for m = 1 and the second part representing the minimum value of the function:

$$\begin{aligned} k_{\sigma,p} &= \frac{\alpha^2(\alpha^2+\theta)+\gamma}{\alpha^2(1+\delta)} && \text{for } \alpha \le \sqrt[4]{\gamma} \\ k_{\sigma,p} &= \frac{2\sqrt{\gamma}+\theta}{(1+\delta)} && \text{for } \alpha > \sqrt[4]{\gamma} \end{aligned} \tag{12}$$

Eq. (12) represents the analytical solution for the buckling coefficient of a longitudinally stiffened plate, taking into account the torsional stiffness of trapezoidal stiffeners.

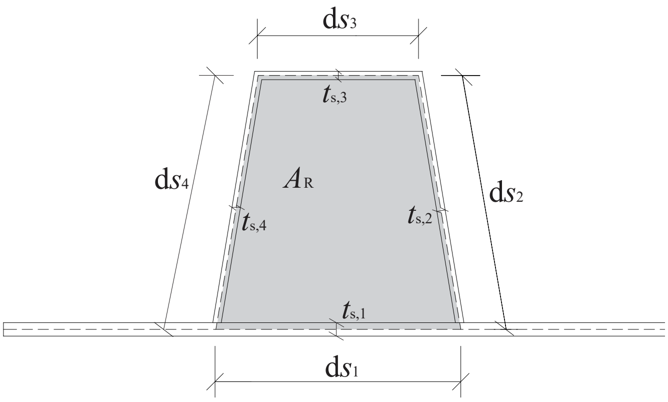


Figure 2 Cross-section properties of a trapezoidal stiffener

3 Numerical determination of $\sigma_{cr,p}$

Linear plate buckling analysis (LBA) was performed to obtain the critical plate buckling stress for global buckling verification, namely $\sigma_{cr,p}$. The calculations were performed using the finite element software ABAQUS [10], and shell elements of type S4R were selected. The direct stresses were applied constantly on the edges of the plate and the stiffeners in the longitudinal direction. Figure 3 shows the simulated boundary conditions and the schematic loading.

The panels were stiffened with four closed trapezoidal stiffeners. To vary the relative stiffness of the stiffeners, the lower and upper widths of the trapezoidal stiffeners were kept constant at 300 and 150 mm, respectively, and the thickness and height of the trapezoidal stiffeners were varied. The stiffeners were arranged so that the width of all subpanels was the same. The geometry and arrangement of the stiffeners are shown in Figure 4.

The following input parameters were varied:

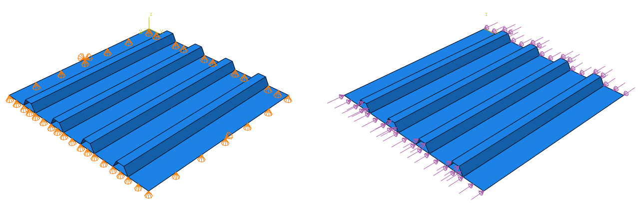
- Slenderness of the panels (global): b/t = from 22 to 533
- Aspect ratios of the panels: α = a/b = 1.0; 1.5; 2.0; 3.0
- Relative bending stiffness of the stiffeners $\gamma_{sl,i}^*$ = 25; 50; 80; 110; 150

The relative bending stiffness of the stiffeners $\gamma_{sl,i}^*$ is defined according to FprEN 1993-1-5 [7] by:

$$\gamma_{sl,i}^* = \frac{EI_{sl,i}^*}{bD} \tag{13}$$

where $I_{sl,i}^*$ is the second moment of area of one stiffener for out-of-plane bending, its cross-section including a participating width of the plate of 10t each side of stiffener-to-plate junction.

The global buckling mode was automatically identified by exporting the deformation from the linear buckling analysis performed in Abaqus to Matlab. In Matlab, the coordinates of the sub-panels and stiffeners were identified and the out-of-plane deformation of all sub-panels and stiffeners was analysed. The deformation of the stiffeners was used as a criterion for determining the global buckling mode. If the deformation was greater than 0.5, the buckling mode was defined as the global buckling mode. More information on the methodology of the global buckling identification can be found in [11].



a) b)

Figure 3 Numerical model for LBA: a) boundary conditions and b) lading [6, 11]

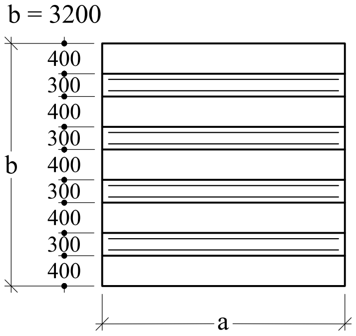


Figure 4 Dimensions and arrangement of the stiffeners in panels of the parametric study [6, 11]

4 Comparison of the analytical calculation with GMNIA results

The new design approach proposed by Pourostad and Kuhlmann [6] and included in the second-generation Eurocode [7] for longitudinally stiffened panels with closed-section stiffeners subjected to compression stresses can be summarised as follows:

- When $\alpha \geq 2$; the reduction factor for plate-like buckling ρ_p has to be determined according to the Müller curve or 12.4 (5) from [1].
- When $\alpha < 2$; the reduction factor for plate-like buckling ρ_p is determined according to the Winter curve.
- The torsional stiffness of closed-section stiffeners can be taken into account when calculating the critical plate buckling stress.
- The reduction factor for column-like behaviour χ_c is determined according to EN 1993-1-5 [1].
- Interpolation should be performed between the reduction factor for plate-like buckling and the reduction factor for column-like buckling to determine the final reduction factor ρ_c according to Eqs. (14, 15).

$$\rho_c = \chi_c + f(\rho_p - \chi_c) \tag{14}$$

$$f = V\left(\ln\left(\frac{\sigma_{cr,p}}{\sigma_{cr,c}}\right)\right)^P \quad \text{but} \quad 0 \leq f \leq 1 \tag{15}$$

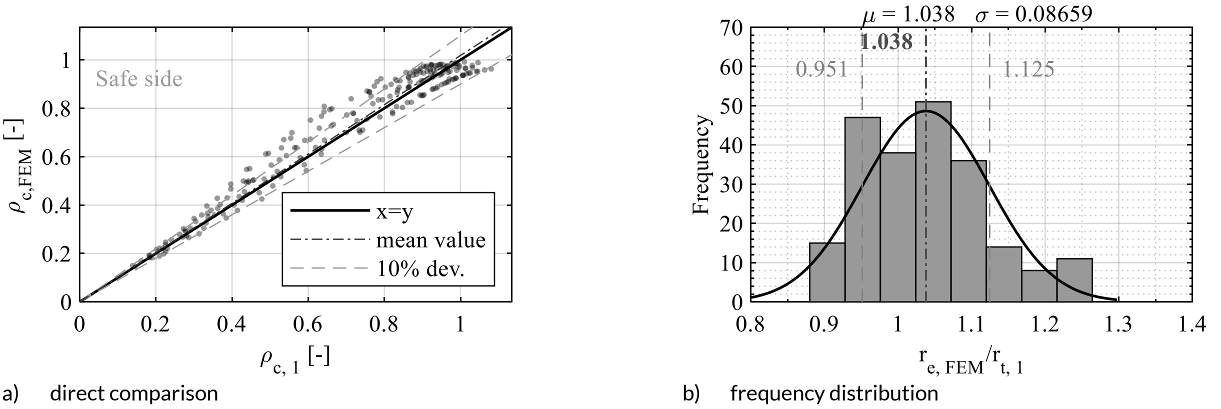
where $\sigma_{cr,p}$ and $\sigma_{cr,c}$ are the elastic critical plate buckling stress and the column buckling stress, respectively. For stiffened panels with closed-section stiffeners under longitudinal stresses, the parameters V and P are defined as follows:

$$V = \left(\bar{\lambda}_p + 1\right)^{-2/3} \quad \text{and} \quad P = 1.0 \tag{16}$$

A numerical parametric study based on a geometrically and materially nonlinear analysis with initial imperfections (GMNIA) was performed by Pourostad and Kuhlmann [6] for a wide range of longitudinally stiffened plates. The procedure and results of the study are described in detail in [6]. In this paper, the results of the numerical parametric study are compared with the previously described new design approach from [7]. For this purpose, two different methods were used to determine the elastic critical plate buckling stress $\sigma_{cr,p}$, namely the new analytical determination described in section 2.2 and the numerical determination described in section 3.

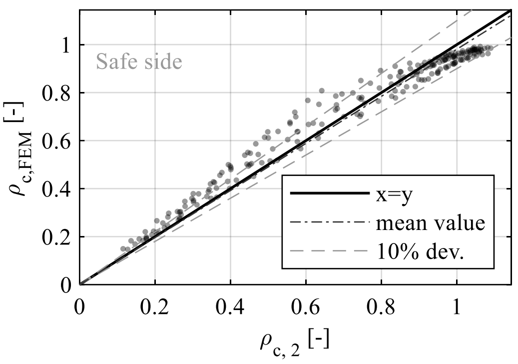
The direct comparison of the loading factors for both methods is shown on the left side of Figures 5 and 6. The numerically obtained loading factor $\rho_{c,FEM}$ is compared with the loading factor calculated using the new design approach, namely $\rho_{c,1}$ and $\rho_{c,2.}$ To achieve $\rho_{c,1}$, the elastic critical plate buckling stress was calculated with the analytical equation from section 2.2, to achieve $\rho_{c,2}$, the elastic critical plate buckling stress was determined numerically using a linear buckling analysis as in section 3. In both cases, the elastic critical column buckling stress was determined according to EN 1993-1-1 [12].

The statistical analysis of the data is represented by histogram diagrams on the right-hand side of Figures 5 and 6. These diagrams include the standard deviation and mean value of the data as well as the vertical lines showing the standard deviation, mean value plus and minus the standard deviation. The analysis shows good agreement with the numerical results for both methods. Determining the resistance of stiffened panels based on the critical stresses through an analytical formula results in smaller standard deviations and mean values compared to the linear buckling analysis.

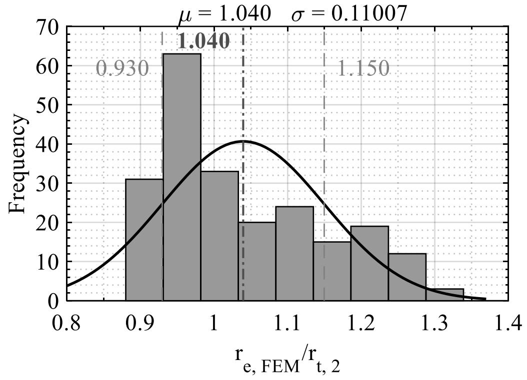


a) direct comparison
b) frequency distribution

Figure 5 Comparison of the numerical results with the results of the new design approach where the elastic critical plate buckling stress is determined analytically according to section 2.2



a) direct comparison



b) frequency distribution

Figure 6 Comparison of the numerical results with the results of the new design approach where the elastic critical plate buckling stress is determined numerically according to section 3

5 Conclusions and outlook

Recent research studies have shown that the positive effects of the torsional stiffness of closed-section longitudinal stiffeners can be taken into account when calculating the ultimate resistance of plates by using alternative interpolation functions. Therefore, this paper presents a new analytical equation for the estimation of the elastic critical buckling stress for longitudinally stiffened plates taking into account the torsional inertia of the stiffeners. It allows the estimation of the plate buckling stress without the use of numerical tools.

The application of the analytical formula to determine the critical plate buckling stresses results in a similar ultimate resistance of panels compared to the complicated and expensive alternative using linear buckling analysis. However, the current formula is proposed only for cases with more than three uniformly distributed stiffeners. It is necessary to extend the scope of the formula to cases with one or two stiffeners.

Acknowledgements

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